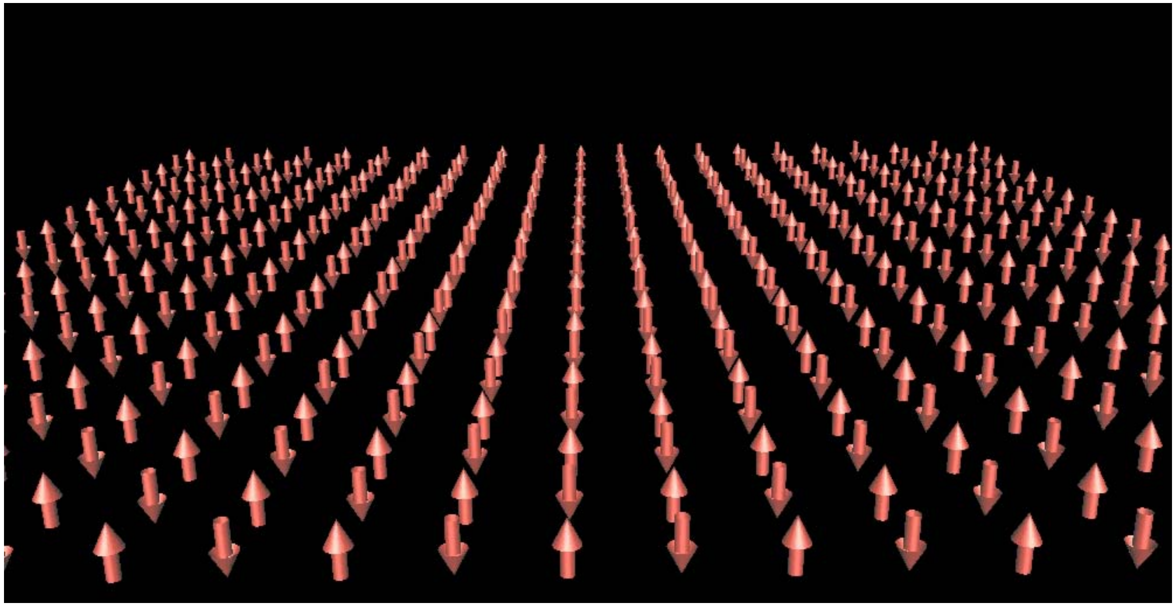
Deconfined Quantum Critical Points

Leon Balents

T. Senthil, MIT
A. Vishwanath, UCB
S. Sachdev, Yale
M.P.A. Fisher, UCSB



Outline

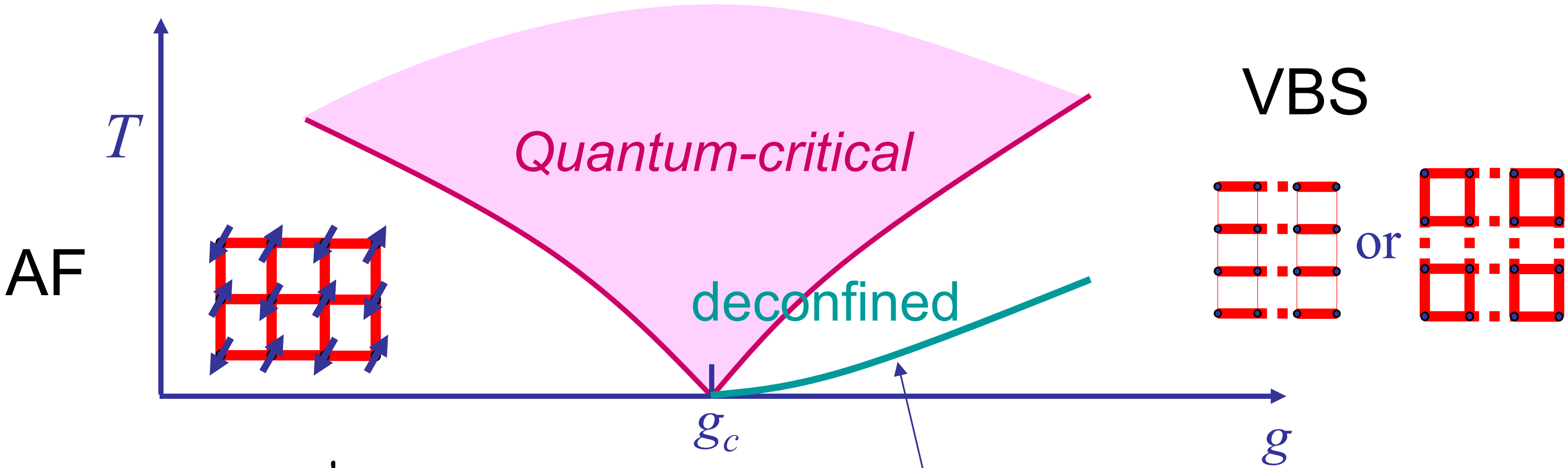
- Introduction: what is a DQCP
- “Disordered” and VBS ground states and gauge theory
- Gauge theory defects and magnetic defects
- Topological transition
- Easy-plane AF and Bosons

What is a DQCP?

- Exotic QCP between two *conventional* phases
- Natural variables are *emergent, fractionalized degrees of freedom* – instead of order parameter(s)
    - *"Resurrection"* of failed U(1) spin liquid state as a QCP
- *Violates Landau rules* for continuous CPs
- Will describe particular examples but applications are much more general

    - c.f. Subir's talk

Deconfined QCP in 2d s=1/2 AF



$$\hat{n} = z^\dagger \vec{\sigma} z$$

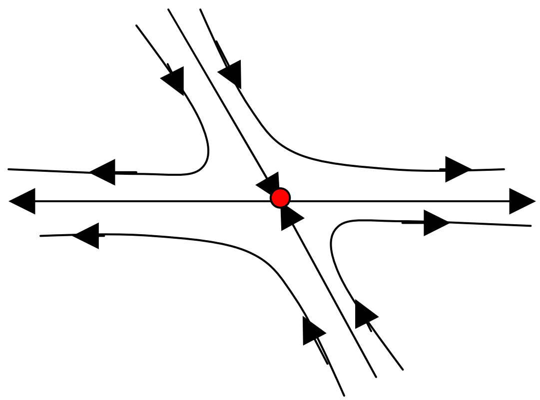
"spinons"

small confinement scale since 4-monopole fugacity is "dangerously irrelevant "

$$\mathcal{L} = \left| \left(\partial_\mu - iA_\mu\right) z_\alpha \right|^2 + s|z|^2 + u\left(|z|^2\right)^2 + \kappa\left(\epsilon_{\mu\nu\kappa}\partial_\nu A_\kappa\right)^2$$

Pictures of Critical Phenomena

- Wilson: RG

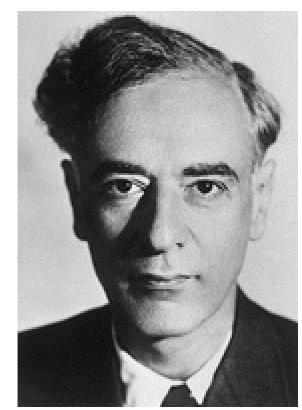


scale invariant field theory with 1 relevant operator

- Landau-Ginzburg-Wilson:

$$F = \int d^d x \, |\nabla \psi|^2 + r|\psi|^2 + u|\psi|^4$$

expansion of free energy (action) around disordered state in terms of order parameter

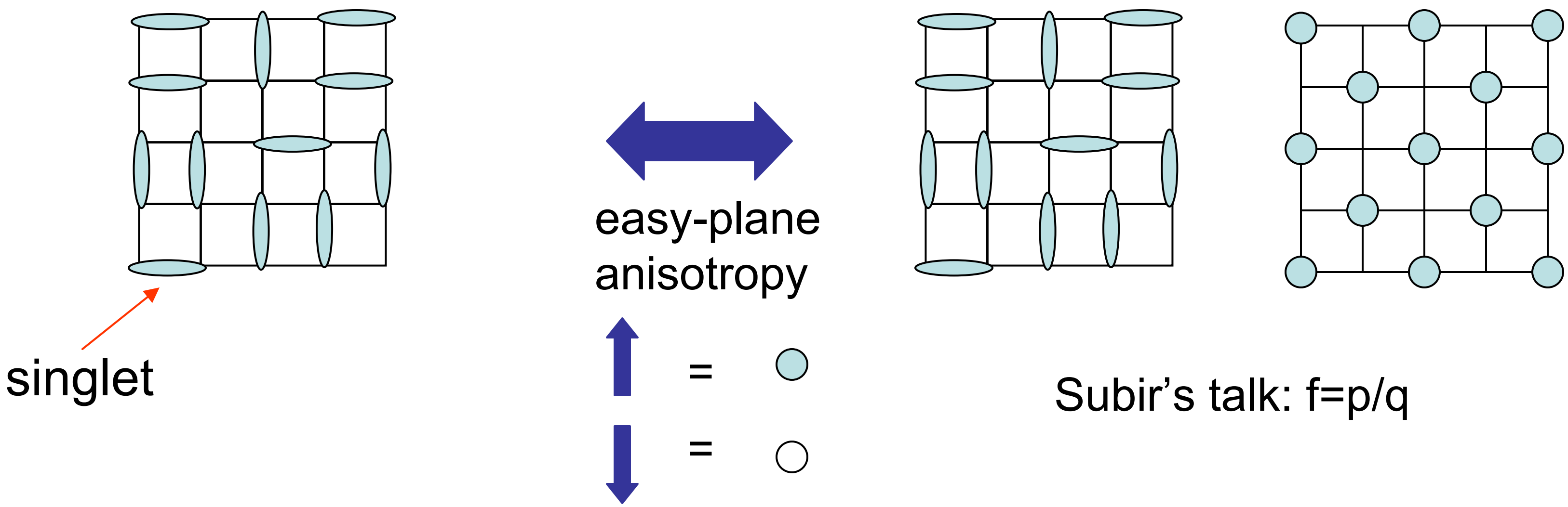


Systems w/o trivial ground states

- Nothing to perform Landau expansion around!

- s=1/2 antiferromagnet

- bosons with non-integer filling, e.g. f=1/2



- Any replacement for Landau theory *must* avoid unphysical "disordered" states

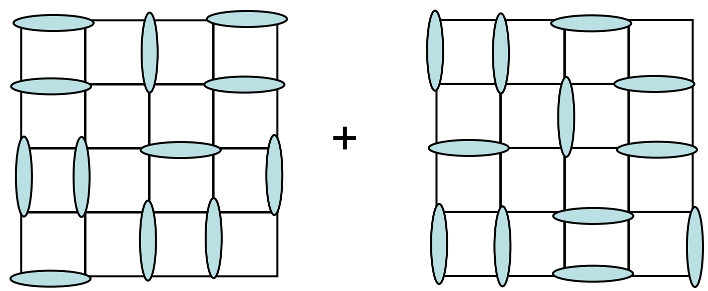
Spin Liquids

Anderson…

- Non-trivial *spin liquid* states proposed
  - -U(1) spin liquid (uRVB)

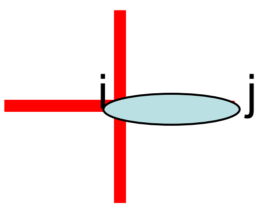
Kivelson, Rokhsar, Sethna, Fradkin
Kotliar, Baskaran, Sachdev, Read
Wen, Lee…

$$|\Psi\rangle =$$



+ …

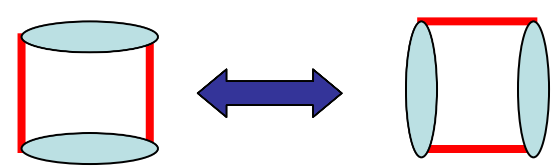
- Problem: described by *compact* U(1) gauge theory (= dimer model)



$$E_{ij} = \begin{cases} +1 & i \in A \\ -1 & i \in B \end{cases}$$

$$\boxed{(\vec{\Delta} \cdot \vec{E})_i = \pm 1}$$

$$H = u \sum_{\langle ij \rangle} E_i^2 - K \sum_{\square} \cos(\epsilon_{ij} \Delta_i A_j)$$

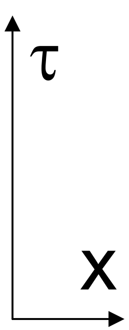


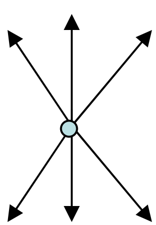
Polyakov Argument

- Compact U(1):
  - E=integer
  - A $\leftrightarrow$ A+2π

$$H = u \sum_{\langle ij \rangle} E_i^2 - K \sum_{\square} \cos(\epsilon_{ij} \Delta_i A_j)$$

- For u$\gg$ K, clearly E_{ij} must order: VBS state
- For K$\gg$ u: E_{ij} *still* ordered due to "monopoles"





$$\vec{\Delta} \cdot \vec{B} = 2\pi Q \delta(x) \delta(\tau)$$ flux changing event

"confinement" (Polyakov): monopole events imply strong flux fluctuations

Dual E field becomes concentrated in lines $[E_i, A_i] = i$

Monopoles and VBS

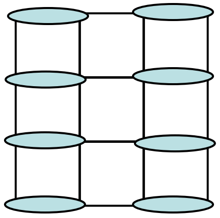
- Unique for s=1/2 system: $(\vec{\Delta} \cdot \vec{E})_i = \pm 1$
- Single flux carries discrete translational/rotational quantum numbers: "monopole Berry phases"

  Haldane, Read-Sachdev, Fradkin

  - only **four-fold** flux creation events allowed by square lattice symmetry
  - *single* flux creation operator $\psi^\dagger$ serves as the VBS order parameter $\psi \sim \psi_{VBS}$

  Read-Sachdev

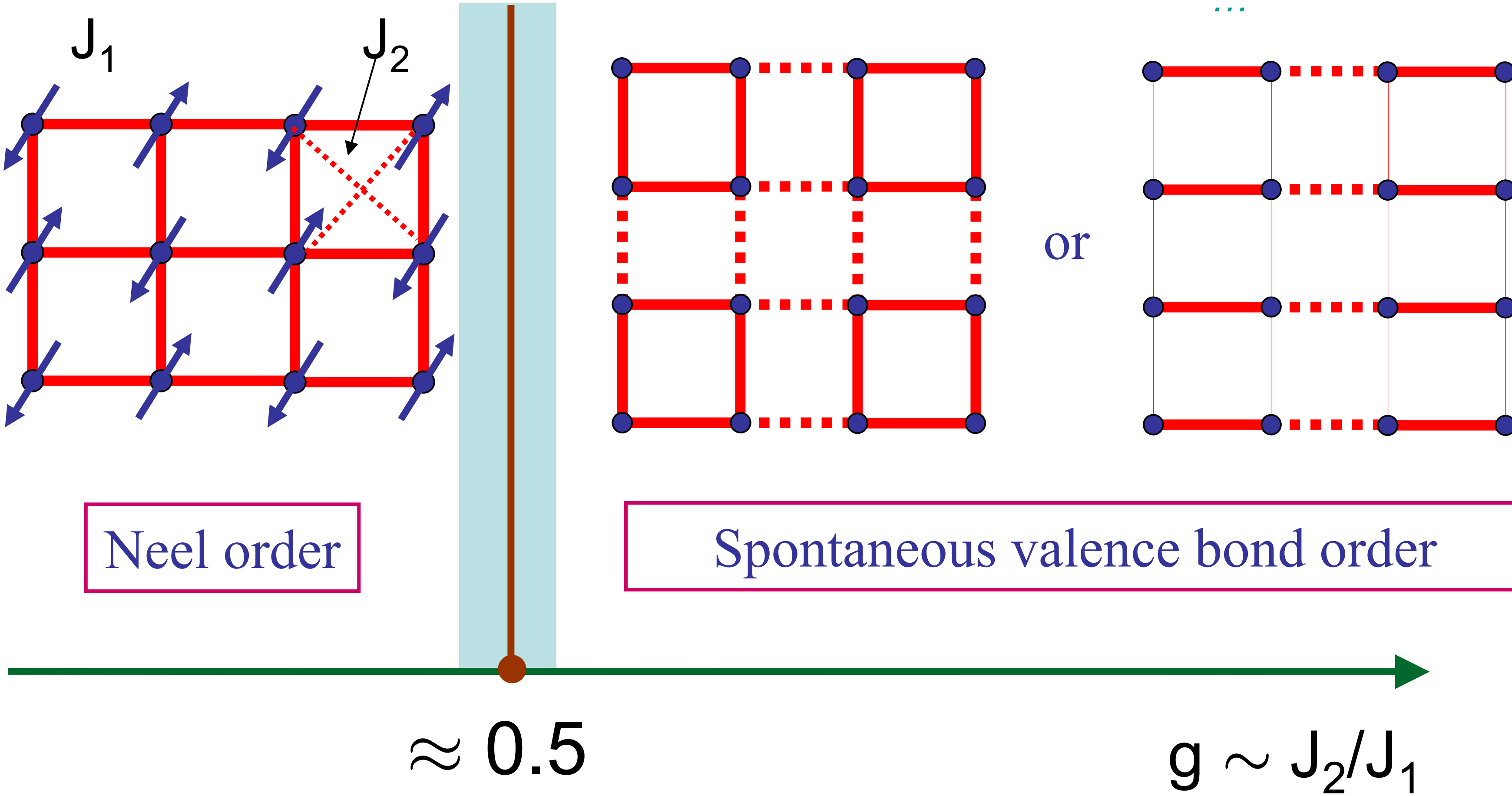
- For pure U(1) gauge theory, *quadrupling* of monopoles is purely quantitative, and the Polyakov argument is unaffected:
  - U(1) spin liquid is generically unstable to VBS state due to monopole proliferation



Neel-VBS Transition

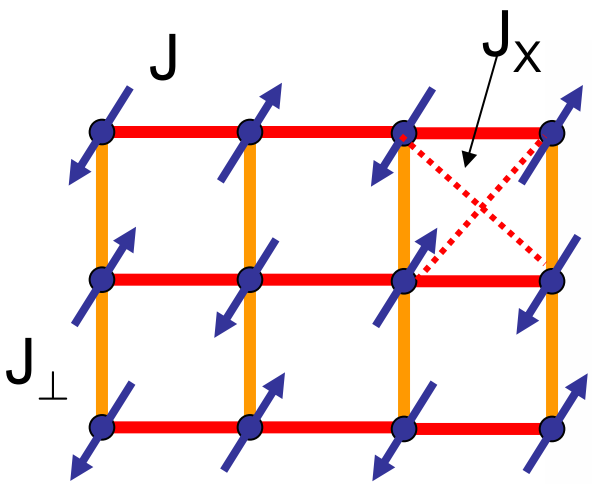
Gelfand *et al*
Kotov *et al*
Harada *et al*
…

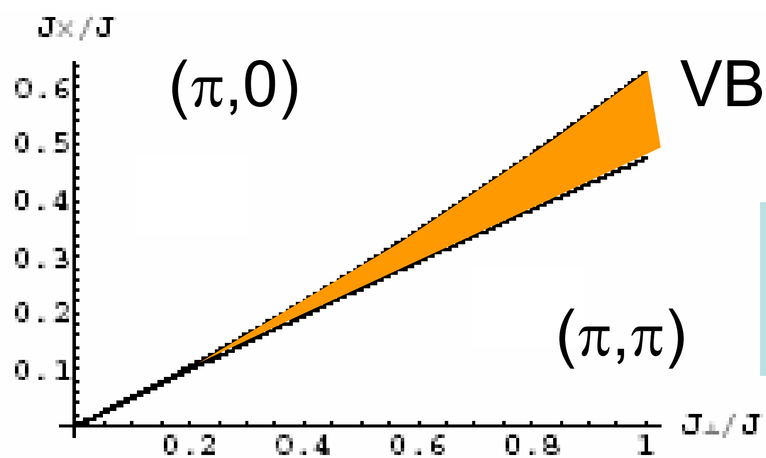
J_1-J_2 model



• Question: Can this be a continuous transition, and if so, how?
- Wrong question: Is it continuous for particular model?

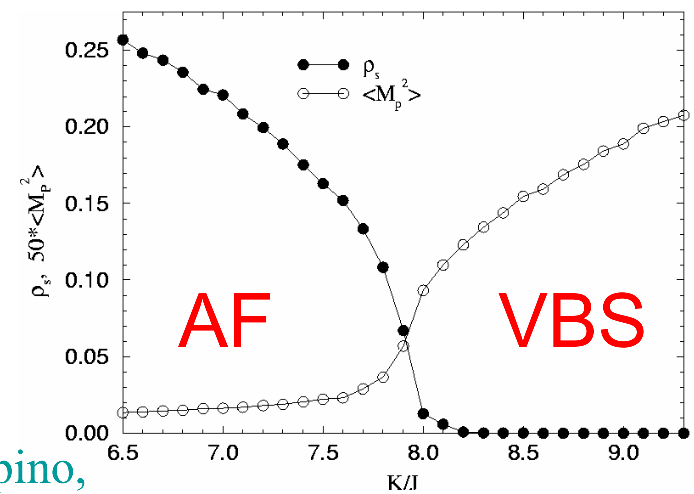
Models w/ VBS Order





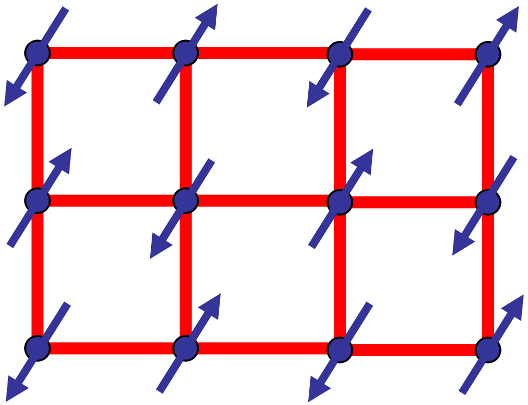
Oleg Starykh and L.B.
cond-mat/0402055

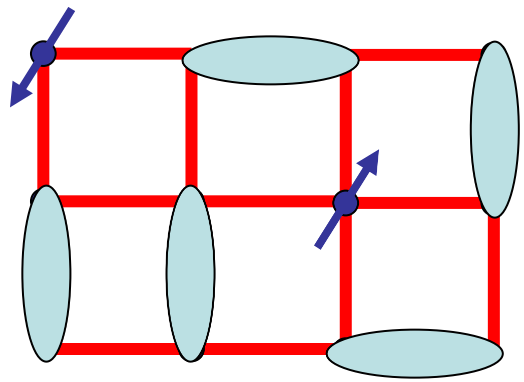
$$H = 2J \sum_{\langle ij \rangle} S_i^x S_j^x + S_i^y S_j^y$$
$$-K \sum_{ijkl \in \square} (S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+)$$

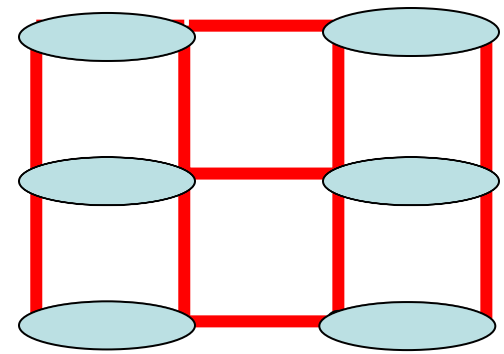


A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino,
B. *Phys. Rev. Lett*. **89**, 247201 (2002)

Spin+Dimer Model=U(1) Gauge Theory

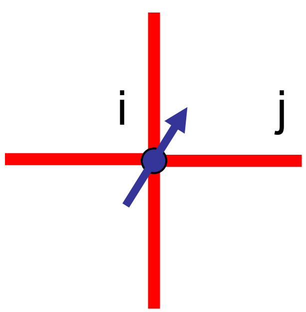


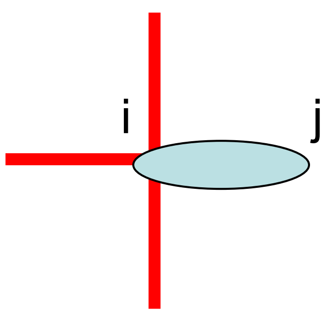




Neel

VBS





$$(\vec{\Delta}\cdot\vec{E})_i = \eta_i(b^\dagger_{i\alpha}b_{i\alpha} - 1)$$

$$b^\dagger_{i\uparrow}b_{i\uparrow} = 1 \qquad E_{ij} = \left(\begin{array}{ll} +1 & i \in A \\ -1 & i \in B \end{array} \right.$$

$b^\dagger_{i\alpha}$ creates *spinon*

CP^1 U(1) gauge theory

spinon

- Some manipulations give: $\vec{n} \sim z_\alpha^\dagger \vec{\sigma}_{\alpha\beta} z_\beta$

$$\mathcal{L} = |\left(\partial_\mu - iA_\mu\right) z_\alpha|^2 + s|z|^2 + u\left(|z|^2\right)^2 + \kappa\left(\epsilon_{\mu\nu\kappa}\partial_\nu A_\kappa\right)^2$$

+ quadrupled monopoles

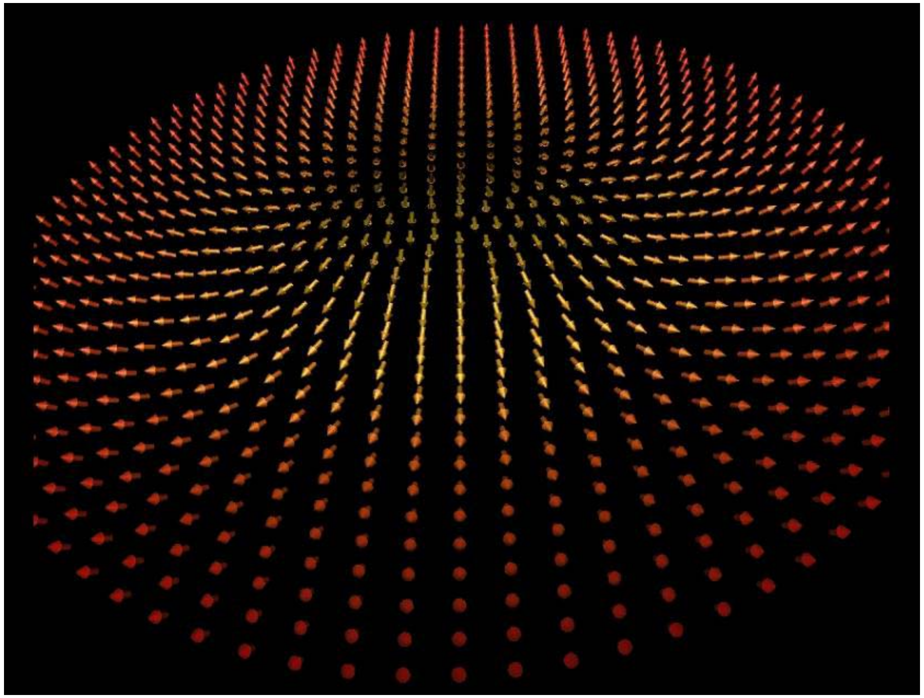
- Phases are completely conventional:
  - -s<0: spinons condense: Neel state $\vec{n} \sim \langle z_\alpha^\dagger \rangle \vec{\sigma}_{\alpha\beta} \langle z_\beta \rangle$
  - -s>0: spinons gapped: U(1) spin liquid unstable to VBS state
  - -s=0: QCP?
- What about monopoles? "Flux quantization"
  - -In Neel state, flux $\pm$ 2π is *bound to skyrmion*
  - -Monopole is bound to "hedgehog"

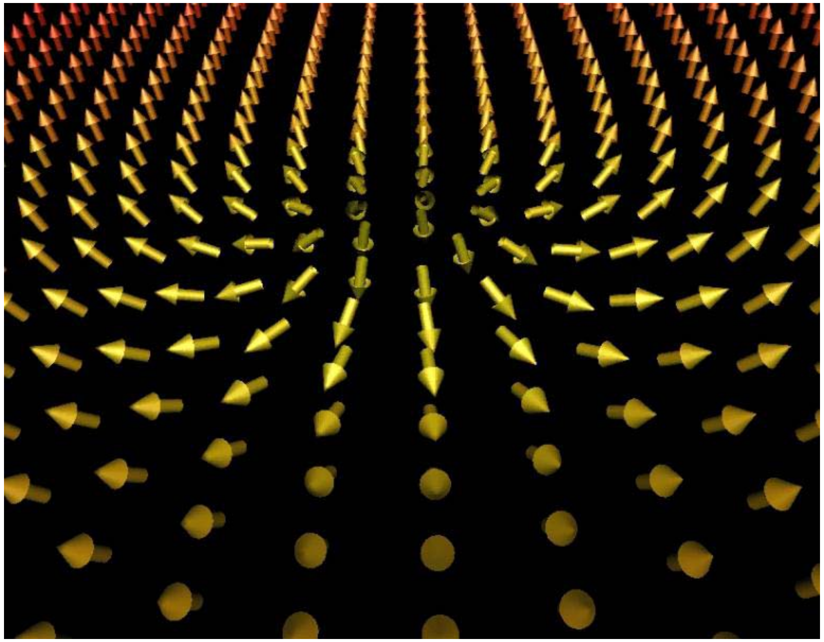
Skyrmions

- Time-independent topological solitons – bound to flux

Integer "index" $Q = \dfrac{1}{4\pi}\int d^2r\, \hat{n}\cdot\partial_x\hat{n}\times\partial_y\hat{n} = \Phi/2\pi$

conserved for smooth configurations



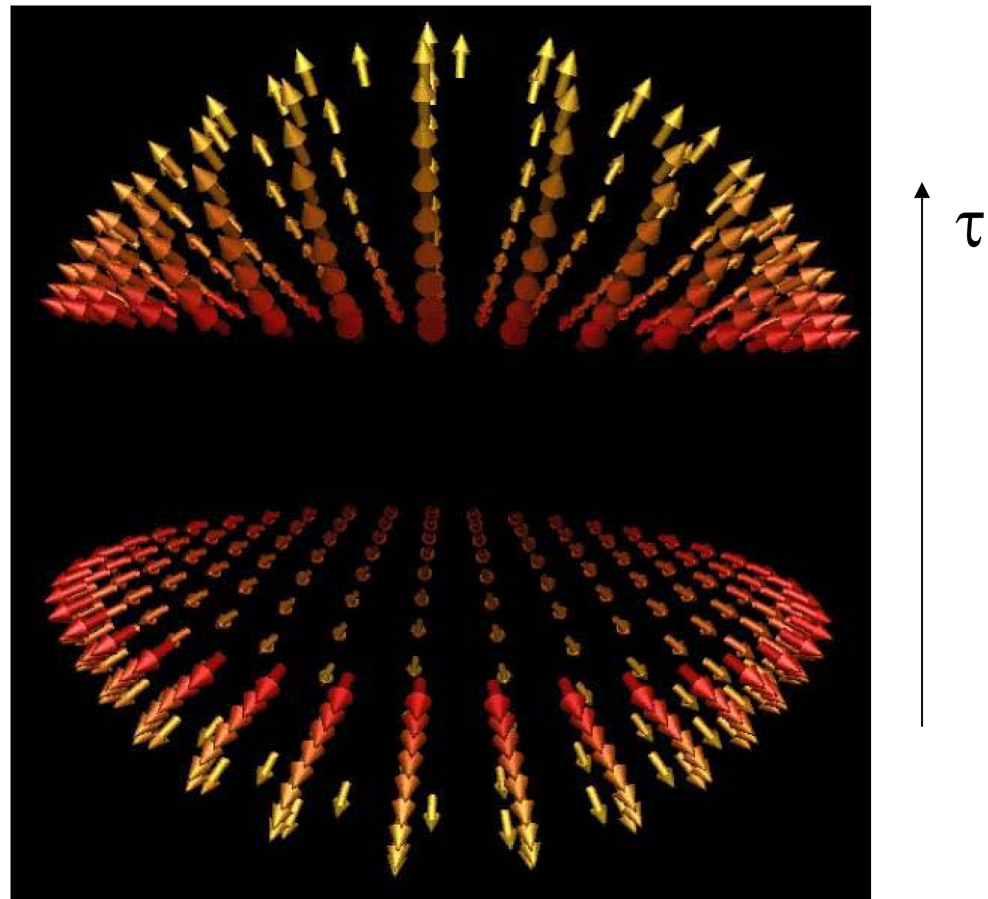


observed in QH Ferromagnets

Hedgehogs

•Monopole is bound to a "hedgehog" action $\sim \rho_s$ L in AF

- singular at one space-time point but allowed on lattice

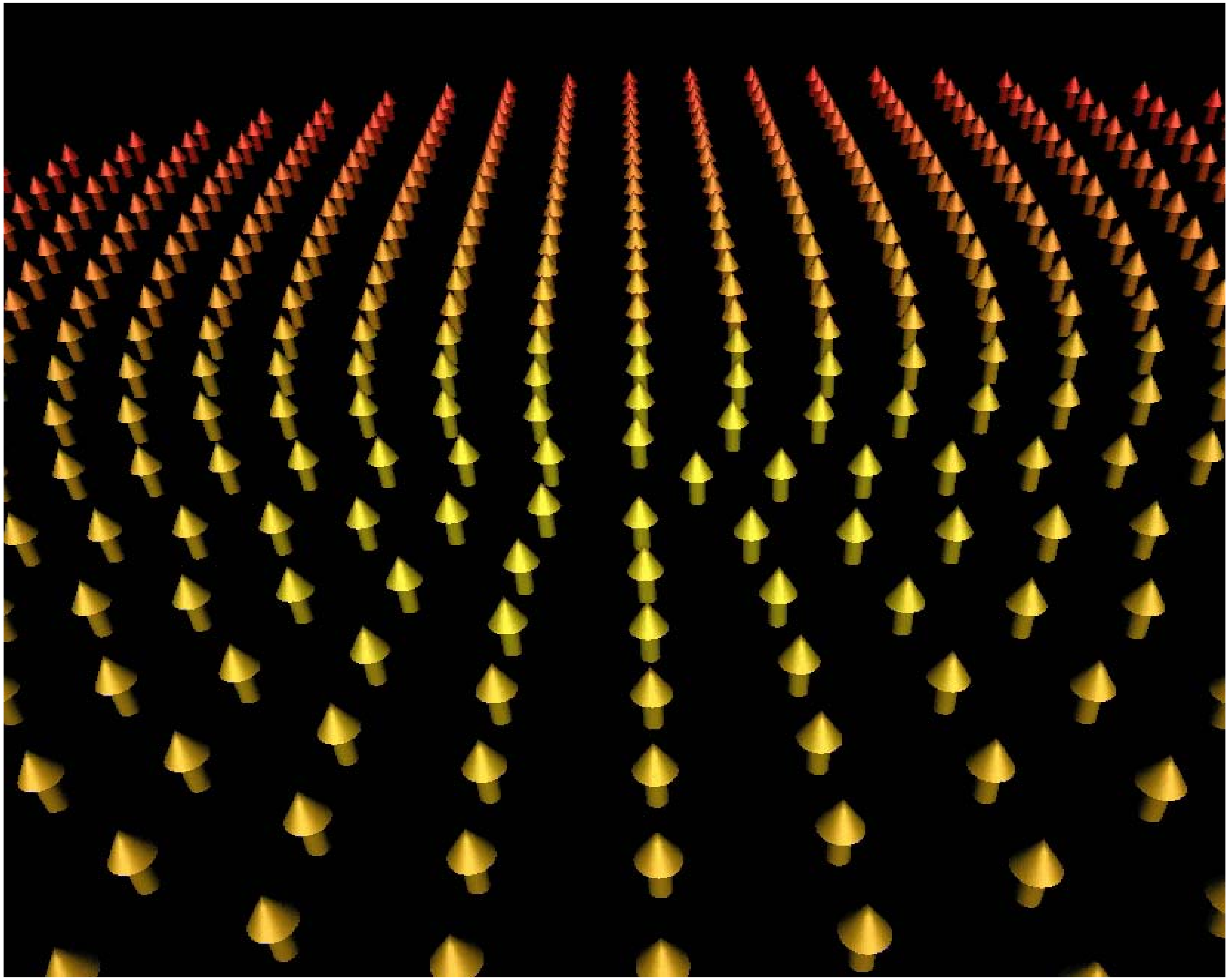


Hedgehogs=Skyrmion Creation Events

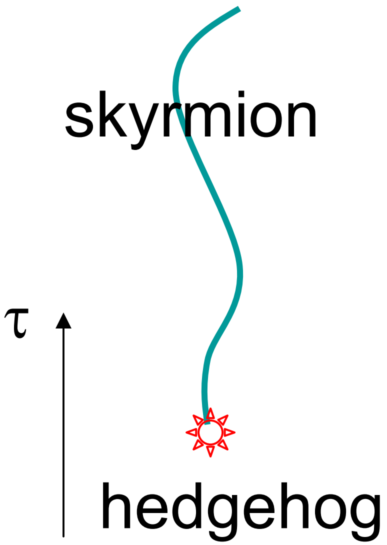
$$|Q = 1\rangle = \psi^\dagger |Q = 0\rangle$$

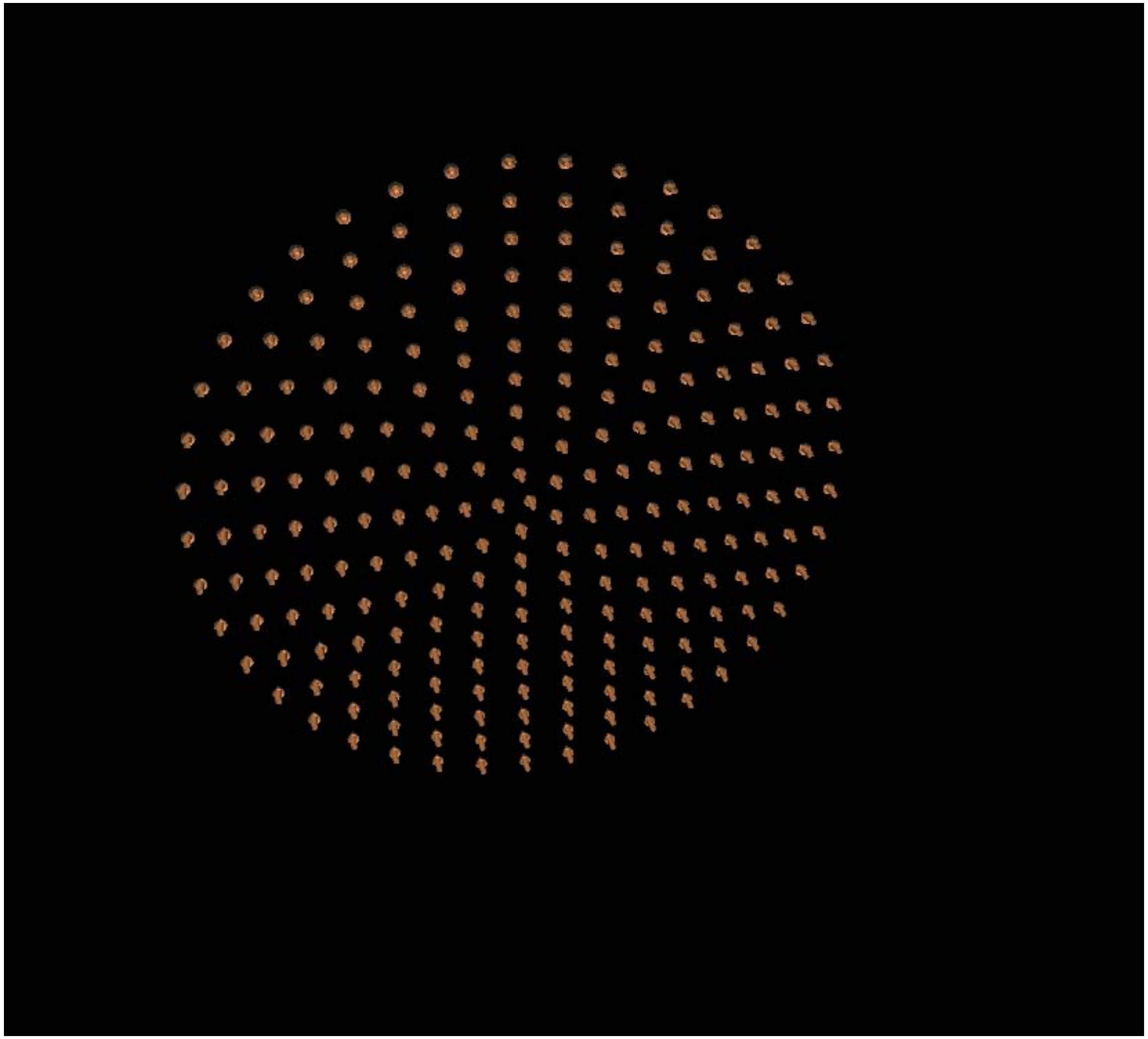
$\Delta Q = +1$

- note "singularity" at origin



Hedgehogs=Skyrmion Creation Events





Fugacity Expansion

- Idea: expand partition function in number of hedgehog events:

$$Z = Z_0 + \int_{r_1} \lambda(r_1) Z_1[r_1] + \frac{1}{2}\int_{r_1,r_2} \lambda(r_1)\lambda(r_2) Z_2[r_1,r_2] + \cdots$$

-λ = *quadrupled* hedgehog *fugacity*
-Z_0 describes "hedgehog-free O(3) model"

- Kosterlitz-Thouless analogy:
  - λ "irrelevant" in AF phase
  - λ "relevant" in PM phase
  -Numerous compelling arguments suggest λ is *irrelevant* at QCP (quadrupling is crucial!)

Topological O(3) Transition

• Studied previously in classical O(3) model with hedgehogs forbidden by hand (Kamal+Murthy. Motrunich+Vishwanath)

- Critical point has modified exponents (M-V)

$$\langle \vec{N}_r \cdot \vec{N}_0 \rangle \sim \frac{1}{r^{1+\eta}}$$

$\eta_{O(3)} \approx .03$ $\eta_{TO(3)} \approx .6\text{-}.7$

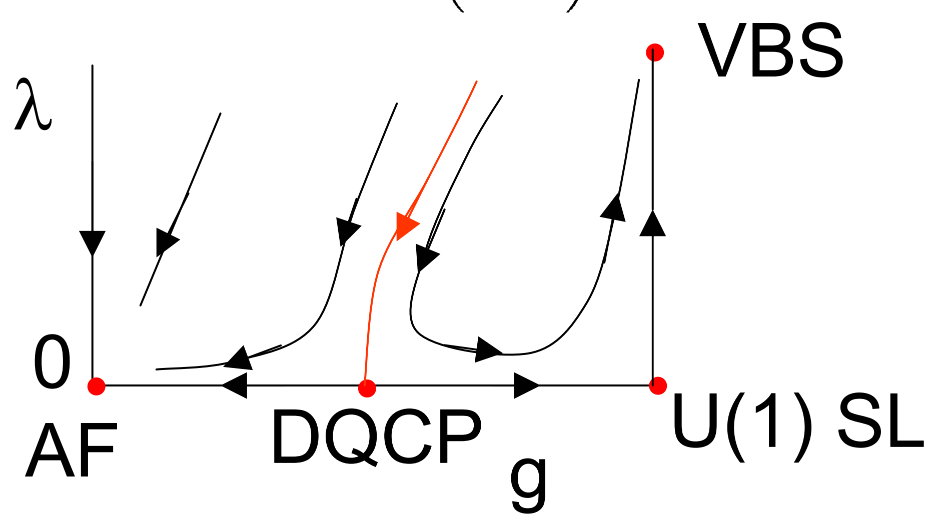
$1/T_1 \sim T^{\eta}$

very broad spectral fnctns

- Same critical behavior as monopole-free CP^1 model

$$\mathcal{L} = |\left(\partial_\mu - iA_\mu\right) z_\alpha|^2 + s|z|^2 + u\left(|z|^2\right)^2 + \kappa\left(\epsilon_{\mu\nu\kappa}\partial_\nu A_\kappa\right)^2$$

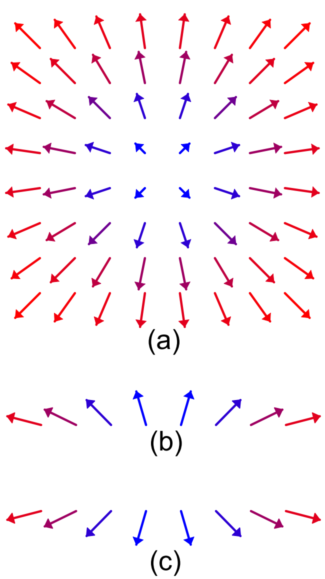
• RG Picture:



Easy-Plane Anisotropy

e.g. lattice bosons

- Add term $\Delta S = v \int d^3 r \, n_z^2$ $\qquad n^+ \sim e^{i\phi}$
- Effect on Neel state
  - -Ordered moment lies in X-Y plane
  - -Skyrmions break up into *merons*



$$\oint \vec{\nabla}\phi \cdot d\vec{\ell} = 2\pi$$

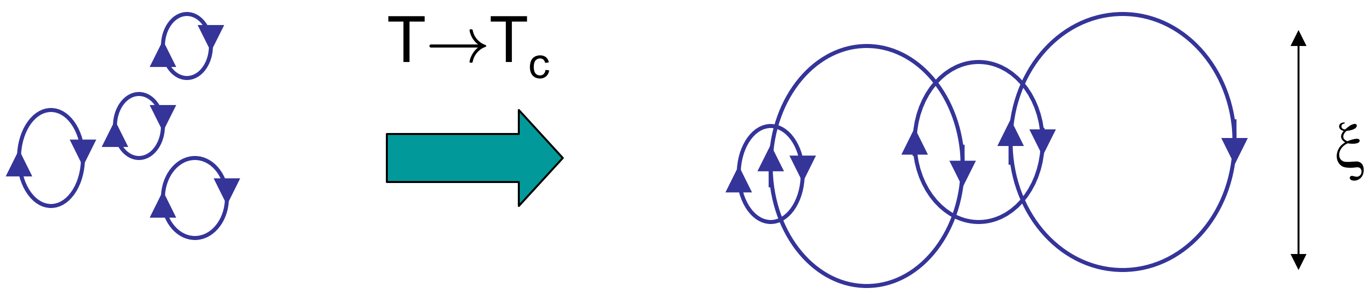
two “flavors” of vortices with “up” or “down” cores

$$n^+ = z_1^* z_2$$

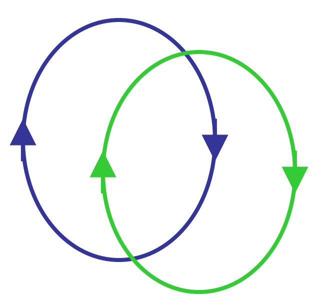
- vortex/antivortex in z_1/z_2

Vortex Condensation

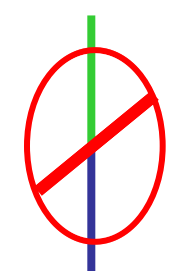
- Ordinary XY transition: proliferation of vortex loops
  - Loop gas provides useful numerical representation



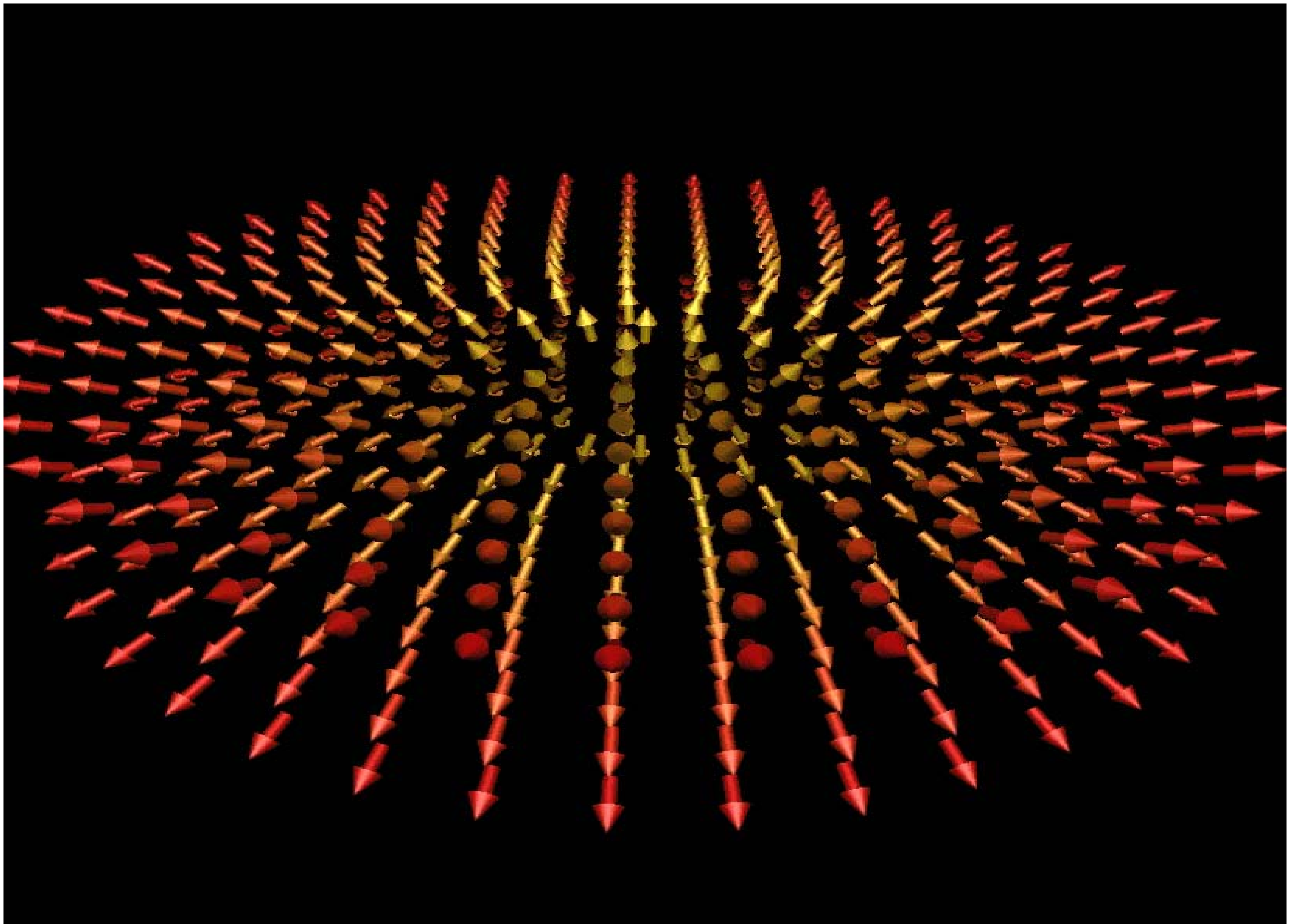
- Topological XY transition: proliferation of two distinct types of vortex loops



Stable if “up” meron does not tunnel into “down” meron



Up-Down Meron Tunneling= Hedgehog

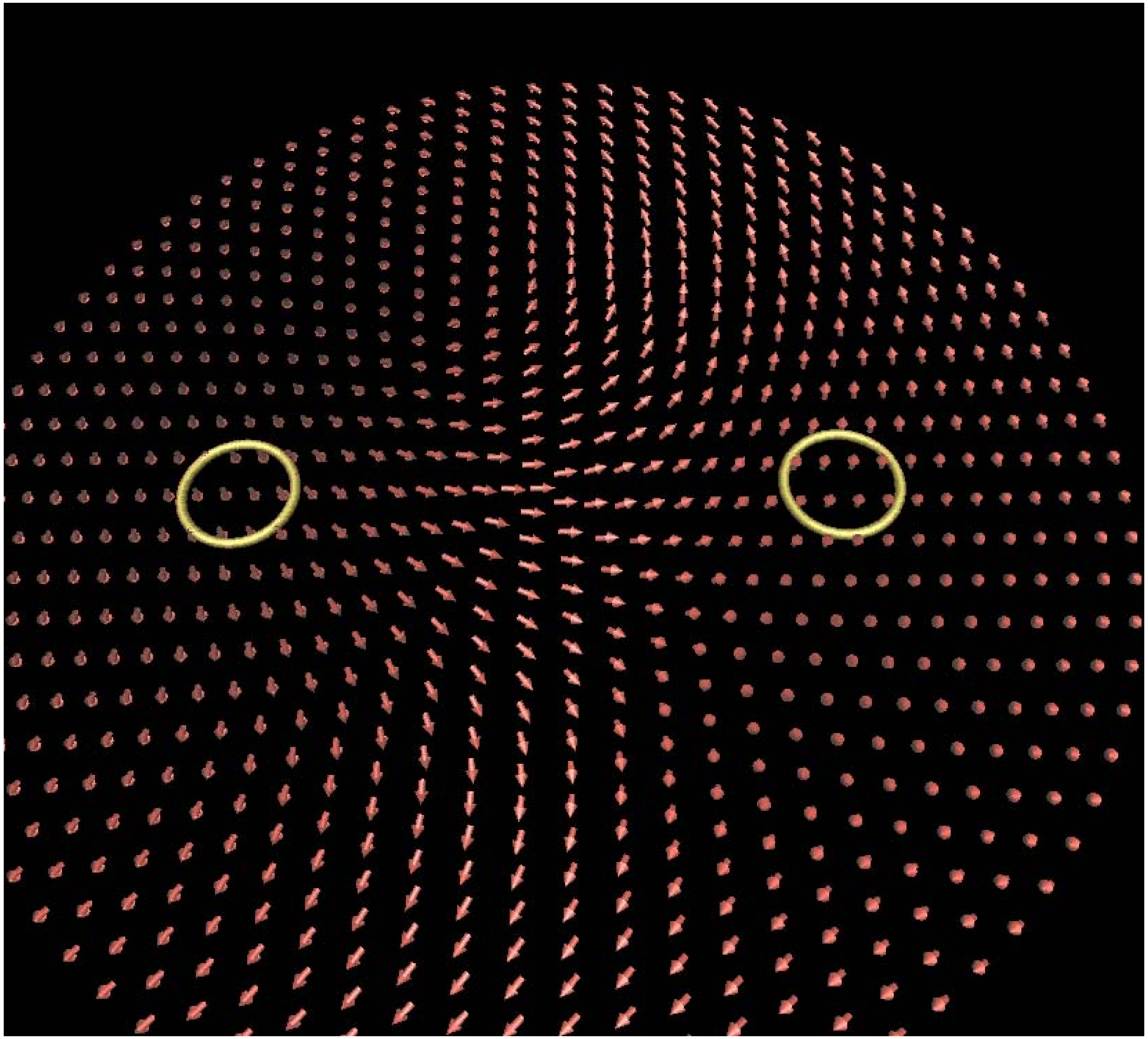


Up/Down Meron Pair = Skyrmion

$$n_1 + in_2 = 2w/(1 + |w|^2)$$

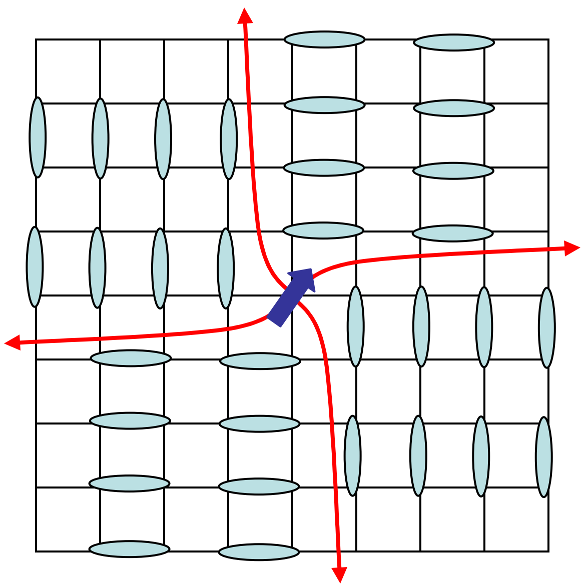
$$n_3 = (1 - |w|^2)/(1 + |w|^2)$$

$$w = \frac{z - a}{z - b}$$



VBS Picture

Levin, Senthil



- Discrete Z_4 vortex defects carry spin ½
  - -Unbind as AF-VBS transition is approached

- Spinon fields z^*_α create these defects

Implications of DQCP

- Continuous Neel-VBS transition exists!
- Broad spectral functions $\eta \sim 0.6$
  – Neutron structure factor
  $$\chi(k_i,\omega) \sim \frac{1}{k^{2-\eta}} F\left(\frac{\omega}{ck}, \frac{\hbar\omega}{k_B T}\right)$$
  – NMR $1/T_1 \sim T^{\eta}$
- Easy-plane anisotropy
  – application: Boson superfluid-Mott transition?
  – self-duality
    - reflection symmetry of T>0 critical lines
    - Same scaling of VBS and SF orders
  – Numerical check: anomalously low VBS stiffness

Conclusions

- Neel-VBS transition is the first example embodying two remarkable phenomena
  - Violation of Landau rules (confluence of two order parameters)
  - Deconfinement of fractional particles
- Deconfinement at QCPs has much broader applications - e.g. Mott transitions

The David and Lucile Packard Foundation

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